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Stability of flux vacua in the presence of charged black holes

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Abstract: In this letter we consider a charged black hole in a flux compactification of type IIB string theory. Both the black hole and the fluxes will induce potentials for the complex structure moduli. We choose the compact dimensions to be described locally by a deformed conifold, creating a large hierarchy. We demonstrate that the presence of a black hole typically will not change the minimum of the moduli potential in a substantial way. However, we also point out a couple of possible loop-holes, which in some cases could lead to interesting physical consequences such as changes in the hierarchy.

Keywords: Black Holes in String Theory, Flux compactifications.

Contents

1. Introduction

Compactifications with non-trivial 3-form fluxes[1] provide an exciting new way to construct phenomenologically interesting stringy models of particle physics and cosmology. These models come to terms with the difficult issue of moduli stabilization [2, 3] and also provide a possible explanation for the hierarchy problem of particle physics [2, 4, 5]. In addition there are important consequences for cosmology. The flux compactifications have lead to a new understanding of the problem of the cosmological constant [6] and can, in addition, incorporate the process of inflation [7].

The four-dimensional effective theory of a flux compactification depends heavily on the value to which the complex structure moduli are fixed. For instance, in the models explaining the hierarchy, the moduli are fixed close to a conifold point. The distance to the conifold singularity then sets the hierarchy [2].

As made explicit in [8, 9], the way complex structure moduli are fixed by fluxes is very analogous to the attractor mechanism [10 − 12] in black hole physics. This raises the important question whether the presence of a charged black hole in a flux background can affect the minimum of the potential for the moduli, and thus affect the hierarchy or other physical properties of the compactification. In this paper we consider such a situation. We choose to study fluxes that fix the moduli close to a conifold point, as in the model explaining hierarchies. In general, adding a black hole breaks all supersymmetries. Nevertheless, we are able to draw some general conclusions.

At string tree-level, the input from the internal geometry to the moduli stabilization physics is, in both cases, governed by the prepotential of the Calabi-Yau. Close to a conifold point, this means that not only the black hole, but also the flux compactification can be

[1] For a recent review with extensive references, see [1].

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described by a matrix model[2]. We will make use of this fact to estimate the shift in the minimum of the potential caused by the black hole.

Our result is that for a generic choice of 3-fluxes the black hole has very small impact on the minimum of the potential. The main reason for this is that the terms in the potential coming from the black hole is suppressed by $\sim 1/(Q^2q^2)$, were Q is integer flux quanta and q is integer black hole D3-brane charge.

We find a few cases when the above argument might be questioned. These include situations with fine-tuned flux quanta, and black holes at the end of their Hawking evaporation, provided the flux quanta and black hole charges are small.

The outline of the paper is as follows. We begin, in section 2, by recalling the relation between the matrix model free energy and the prepotential of a conifold limit of a compact Calabi–Yau. With this prepotential all the interesting attractor phenomena can be studied. Section 3 recalls the relevant material from black hole attractor physics and flux compactifications, always keeping our explicit example in mind. In both cases we find effective four-dimensional potentials for the complex structure moduli. In section 4 we study the combined system and compare the relative importance of the potentials. The paper ends with the conclusions.

2. The prepotential from the matrix model

Below we review the established connection [13,14] between the matrix model and black holes. We do this in order to make the reader think in matrix model terms when we later discuss both black holes and flux compactifications. This unified view on the two systems is fruitful from a conceptual point of view, and will hopefully deepen our understanding of the physics of flux compactifications. Also, we explain in more detail the origin of the non-universal terms, crucial for modelling the embedding of the conifold into a compact Calabi-Yau.

According to [15] there is a direct relation between BPS black holes in 4D type IIB supergravity and topological strings propagating on the Calabi-Yau on which the type IIB ten-dimensional theory is compactified. In [13, 14] this fact was combined with the results of [16] to set up a detailed match between the free energy of the $c = 1$ matrix model[3] and the entropy of these extremal black holes.

We are interested in internal manifolds which has complex structure moduli such that they locally look like deformed conifolds. Not only is this the limit where the matrix model tools are applicable, but it is also the limit used to explain hierarchies in flux compactifications.

Let us review the calculation in [14] — with some more details — of the free energy in the matrix model paying attention to large non-universal terms. These non-universal terms give the main contribution to the entropy, while some of the dependence on the complex

[2]For the black hole case, this correspondence holds to all orders in the string loop expansion as described in [13, 14].

[3]For a nice review of the matrix model, see [17].

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structure moduli is captured by the universal terms. For concreteness we start out with a regulated double well potential given by

$$V(\lambda) = -\frac{\lambda^2}{\alpha'} + A\lambda^4. \tag{2.1}$$

Using N fermions we fill up the Fermi sea to a level μ as measured from the top of the potential. The conifold physics is then described by what is going on near the top of the potential, while the regulating quartic piece rounds off the conifold and makes it part of a Calabi-Yau manifold with finite volume [14]. The details of the regularisation capture the shape of the manifold away from the conifold tip. Our task is then to find an expression for the canonical free energy $F_{MM}(N,\beta)$ for the system and its Legendre transform $\mathcal{F}_{MM}(\mu,\beta)$. To accomplish this we express the free energy and the number of fermions as

$$F_{MM}(N,\beta) = \int^{-\mu} d\varepsilon\varepsilon\rho(\varepsilon), \tag{2.2}$$

where μ is to be substituted for N according to

$$N = \int^{-\mu} d\varepsilon\rho(\varepsilon), \tag{2.3}$$

and, where the integration in energy goes from the bottom of the potential up to the Fermi surface. The density of states is given by

$$\rho(\varepsilon) = \beta \int_{\lambda_-}^{\lambda_+} \frac{d\lambda}{\sqrt{2\left(\varepsilon + \frac{\lambda^2}{\alpha'} - A\lambda^4\right)}}, \tag{2.4}$$

where the integration limits are the shores of the Fermi sea. It is now a simple exercise to compute the free energy and we arrive at

$$\beta\mathcal{F}_{MM}(\mu,\beta) = \frac{1}{\sqrt{\alpha'}}N_0^2 - N_0\mu\beta - \sqrt{\alpha'}(\beta\mu)^2\ln(\mu/\Lambda), \tag{2.5}$$

where

$$\Lambda \sim \frac{1}{A\alpha'^2} \tag{2.6}$$

is an effective cutoff introduced by the quartic piece of the potential. N_0 is the number of fermions needed if we fill the potential all the way up and is given, through Bohr-Sommerfeldt quantization by

$$N_0 \sim \frac{\beta}{A\alpha'^{3/2}}. \tag{2.7}$$

Let us explain in some more detail the origin of the various terms. The last term in expression (2.5) is well known and is simply the standard non-analytic universal contribution to the free energy of the matrix model. In contrast, the first two terms have an analytic dependence on μ and do not play any role in the usual matrix model analysis. Here, however, they are of crucial importance. The second term is a consequence of the relation

$$N_0 = -\left.\frac{\partial\mathcal{F}_{MM}(\mu,\beta)}{\partial\mu}\right|_{\mu=0}, \tag{2.8}$$

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while the first is obtained from

$$\mathcal{F}_{MM}(0,\beta) = -F_{MM}(N_0,\beta) = -\int^0 d\varepsilon\varepsilon\rho(\varepsilon) \sim \frac{1}{\sqrt{\alpha'}\beta}N_0^2. \tag{2.9}$$

Here we have used $\rho(\varepsilon) \sim \beta\sqrt{\alpha'}$ to estimate the bulk density of states. Expressing N_0 in in terms of the parameters of the problem we finally get

$$\beta\mathcal{F}_{MM}(\mu,\beta) = \frac{1}{A^2\alpha'^{7/2}}\beta^2 - \frac{\mu\beta}{A\alpha'^{3/2}}\beta - \sqrt{\alpha'}(\beta\mu)^2\ln(\mu/\Lambda). \tag{2.10}$$

The calculation is performed at zero temperature, but as argued in [13, 14], the relevant temperature of the matrix model should actually be a multiple of the self-dual temperature in order to describe the conifold.[4] It can be shown, however, that the general form of the free energy does not change.

Written in the way above, the free energy of the matrix model provides interesting information about the entropy of four-dimensional black holes. The canonical free energy $F_{MM}(N,\beta)$ is directly proportional to the black hole entropy with the various parameters being related to two sets of electric and magnetic charges. The number of fermions can be associated with an electric charge $q_1 \sim N$. The main contribution to the entropy is given by the analytic piece and is of the form $S \sim N^2$, while the universal non-analytic piece tells us how the entropy varies close to the conifold value N_0. The black hole also have a magnetic charge given by $p^0 \sim \beta$. As argued in [13, 14] we can also turn on another magnetic charge, p^1, which from the matrix model point of view corresponds to deforming the potential by a $1/\lambda^2$ piece.

Furthermore, as discussed in [13, 14], the free energy of the matrix model is directly related to the imaginary part of the prepotential of the four-dimensional supergravity theory. In the next section we will write down the corresponding prepotential and review how the attractor equations obtained from the four-dimensional analysis reproduce known properties of the matrix model and the corresponding black hole. We will also use the same prepotential to accomplish moduli stabilization through a flux compactification. In this way we obtain a mapping between quantities of the matrix model and space time not only in the case of a black hole, but also for flux compactifications.

3. Moduli stabilization in type IIB supergravity

Consider ten-dimensional type IIB supergravity. Neglecting the Chern-Simons term, the bosonic action is given by[5]

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\mathcal{R} - \frac{\partial_M\tau\partial^M\tau}{2(\mathrm{Im}\tau)^2} - -\frac{|G_3|^2}{12\mathrm{Im}\tau} - \frac{|\tilde{F}_5|^2}{4\cdot 5!}\right). \tag{3.1}$$

Here $G_3 = F_3 - \tau H_3$ and $\tau = \frac{i}{g_s} + C_0$ is the axio-dilaton.

[4]Note that the temperature of the black hole in space time is still zero, as explained in [13, 14].

[5]We use the notations of [2].

We will study compactifications of this theory to four dimensions, letting the internal dimensions be a (possibly conformal) Calabi-Yau manifold Y. Specifically we will assume a complex structure moduli space $\mathcal{M}$ of complex dimension one, and that we are close to a conifold point. Let $\{A^I, B_I\}$ $I = 0, 1$ be a symplectic basis of $H_3(Y)$, so that A^1 is the conifold cycle. Furthermore, let $\{\alpha_I, \beta_I\}$ be a basis of $H^3(Y)$ so that, as usual,

$$\oint_Y \alpha_J \wedge \beta^I = \oint_{A^I} \alpha_J = \oint_{B_J} \beta^I = \delta^I_J. \tag{3.2}$$

The periods of the holomorphic 3-form Ω are defined by

$$X^I = \oint_{A^I} \Omega \tag{3.3}$$

$$F_J(X^I) = \oint_{B_J} \Omega. \tag{3.4}$$

Let us work in a Kähler gauge in which $X^0 = V^{1/2}$ and $X^1 = V^{1/2} z$. V is the (unwarped) volume of the Calabi-Yau, and z is the coordinate on $\mathcal{M}$ vanishing at the conifold point.

Close to the conifold the prepotential is given by

$$F = ia_1 (X^0)^2 + a_2 X^0 X^1 + ia_3 \left(X^1\right)^2 \ln \frac{X^1}{X^0}, \tag{3.5}$$

where other terms of order $\mathcal{O}(z^2)$ have been neglected, and the a_i are numerical coefficients depending on the Calabi-Yau geometry. Specifically, $a_3 = -1/4\pi$. Note that this prepotential has exactly the same functional form as the matrix model free energy.

Let us now study in turn how wrapped branes and fluxes behave on this geometry.

3.1 A black hole attractor

The presence of a black hole consisting of wrapped D3-branes generates an effective potential for the complex structure moduli. The potential is induced by the 5-form field strength $\tilde{F}_5$ sourced by the black hole charges. The metric is an unwarped product between a four-dimensional part and a Calabi-Yau part whose complex structure depends on the space-time point.

We write the four-dimensional part of the black hole metric in the form

$$\tilde{g}^{(4)}_{\mu\nu} dx^\mu dx^\nu = -e^{2u(\sigma)} dt^2 + \frac{e^{-2u(\sigma)} c^4 d\sigma^2}{\sinh^4(c\sigma)} + \frac{e^{-2u(\sigma)} c^2}{\sinh^2(c\sigma)} d\Omega^2, \tag{3.6}$$

where $c \to 0$ is the extremal limit. Here σ goes from $-\infty$ (horizon) to 0 (spatial infinity). Furthermore $u \sim c\sigma$ as $\sigma \to -\infty$.

In the notation of [18] the field strength is given by

$$\tilde{F}_5 = \mathcal{F} + * \mathcal{F} = \sin\theta d\theta \wedge d\phi \wedge \Gamma + e^{2u} dt \wedge d\sigma \wedge \hat{\Gamma}. \tag{3.7}$$

Here Γ is a 3-form corresponding to the black hole charge, and $\hat{\Gamma} = *_6 \Gamma$ is its six-dimensional Hodge dual. In particular, if the D3-brane wraps the cycles $A^I(B_I)$ $q_I(p^I)$ times, then $\Gamma = 4\pi^3 (\alpha')^2 (p^I \alpha_I + q_I \beta^I)$ and, consequently,

$$\oint_{S^2 \times A^I} \tilde{F}_5 = ((2\pi)^2 \alpha')^2 p^I \text{ and } \oint_{S^2 \times B_I} \tilde{F}_5 = ((2\pi)^2 \alpha')^2 q_I, \tag{3.8}$$

for any space-like S^2 enclosing the location of the D3-branes.

In the four-dimensional effective action, this field strength gives rise to the term

$$S_{pot} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\frac{|\tilde{F}_5|^2}{4\cdot 5!} = -\frac{1}{2\kappa_{10}^2}\int d\mathrm{Vol}_4\frac{1}{r^4}\oint_Y \Gamma\wedge\hat{\Gamma} = \\ = -\frac{1}{2\kappa_{10}^2}\int d\mathrm{Vol}_4\frac{V_{bh}(z)}{r^4}, \tag{3.9}$$

where we reinserted the usual radial coordinate r. In the case of a BPS black hole, the potential V_{bh} can be obtained via the Gukov-Vafa-Witten (GVW) superpotential $W_{bh} = \int_Y \Omega\wedge\Gamma$ [19] as the usual $\mathcal{N}=2$ scalar potential

$$V_{bh}(z) = e^{K_{bh}}\left(\mathcal{G}_{bh}^{z\bar{z}}D_zW_{bh}D_{\bar{z}}\overline{W}_{bh} + |W_{bh}|^2\right), \tag{3.10}$$

where $\mathcal{G}_{bh}$ is the metric derived from the Kähler potential $K_{bh} = -\ln i\left(\bar{X}^IF_I - X^I\bar{F}_I\right)$ on $\mathcal{M}$. Using equation (3.5), it is straightforward to express W_{bh} and K_{bh} in terms of the geometrical coefficients a_i and the black hole charges. Explicitly, with our gauge choice, we obtain[6]

$$W_{bh}(z) = V^{1/2}(\alpha')^2w(z) \tag{3.11}$$

with

$$w = q_0 - 2ia_1p^0 - a_2p^1 + \left(q_1 - a_2p^0 - ia_3p^1\right)z + ia_3p^0z^2 - 2ia_3p^1z\ln z. \tag{3.12}$$

Furthermore, we have the Kähler potential $K_{bh} = K_{bh}(z,\bar{z})$ given by

$$e^{-K_{bh}} = Vk(z,\bar{z}) = V\left(4a_1 - a_3(z-\bar{z})^2 + 2a_3|z|^2\ln|z|^2\right). \tag{3.13}$$

It is now easy to see that the usual matrix model results are reproduced. If we minimize the potential through $\partial_zV_{bh} = 0$, we find

$$D_zW_{bh} = V^{1/2}\frac{kw_z - k_zw}{k} = 0. \tag{3.14}$$

This is just the attractor equations for the complex structure moduli. Focusing on the black hole corresponding to the undeformed matrix model (it is easy to generalize to the general case), we have $q_0 = p^1 = 0$ and the attractor equations tell us that, to first order in z,

$$q_1 = a_2p^0 - 2a_3p^0x\ln x - a_3p^0x, \tag{3.15}$$

where $z = ix$ and x is real. Note that $z = \frac{X^1}{X^0} \sim i\mu$. This is nothing else than the formula for the number of fermions $q_1 \sim N$ near its critical value given by $p^0 \sim \beta$, when we fill up the Fermi sea towards the top of the potential.

[6] For notational simplicity we ignore factors of π.

3.2 Flux compactifications and hierarchies

Warped geometries have played a crucial role in the construction of realistic phenomenological models. The reasons are twofold. On the one hand the introduction of 3-form fluxes, F_3 and H_3, on the compact manifold works just like the introduction of space time filling D3-branes. These branes appear as point sources on the compact manifold and correspond to deep throats of warped geometry. The warping introduces a relative redshift between various points on the compact manifold which can be used to explain hierarchies of scales.

On the other hand, the fluxes introduce potentials for the complex moduli of the Calabi-Yau manifold. This happens quite analogously to the black hole case. One difference, however, is that the potential now receives contributions not only from the 3-form flux term in the action, but also from the 5-form and the Einstein-Hilbert term. Through the equations of motion, these terms can be rewritten in terms of the fluxes.

The potential part of the effective action becomes [4]

$$S_{pot} = -\frac{1}{2\kappa_{10}^2}\int d\mathrm{Vol}_4 \oint_Y \frac{e^{4A}}{2\mathrm{Im}\tau} G_3 \wedge \left(*_6\overline{G}_3 + i\overline{G}_3\right) = -\frac{1}{2\kappa_{10}^2}\int d\mathrm{Vol}_4 V_f\left(z\right). \tag{3.16}$$

Also in this case, the form of the effective potential for the moduli is governed by the geometry of the internal manifold. It it given by the usual $\mathcal{N} = 1$ scalar potential

$$V_f\left(z\right) = e^{K_f}\left(\mathcal{G}^{A\overline{B}} D_A W_f D_{\overline{B}} \overline{W}_f - 3\left|W_f\right|^2\right). \tag{3.17}$$

where $W_f = \int_Y \Omega \wedge G_3$ again is the GVW superpotential. The indices A, B go over z, τ and the volume modulus ρ. The Kähler potential K_f now also depends on the axio-dilaton and on the volume modulus:

$$K_f = -\ln\left[-i(\tau-\bar{\tau})\right] - 3\ln\left[-i(\rho-\bar{\rho})\right] - \ln\left[-i\oint_Y e^{-4A}\Omega\wedge\overline{\Omega}\right]. \tag{3.18}$$

The coefficient of the ρ term shows that the Kähler potential is of no-scale form, as noticed in [2]. From here it is a straightforward calculation to obtain the behaviour of V_f in terms of the flux quanta and the geometrical parameters a_i. We return to this in the next section.

We choose non-zero fluxes such that

$$\oint_{A^1} F_3 = (2\pi)^2\alpha' P^1 \tag{3.19}$$

$$\oint_{B_1} H_3 = -(2\pi)^2\alpha' Q_1, \tag{3.20}$$

where P^1 and Q_1 are integers. If we do this we end up with a superpotential of the same form as in [2]. Specifically, we have (still ignoring factors of π)

$$W_f\left(z\right) = V^{1/2}\alpha'\left(-a_2P^1 + \left(\tau Q_1 - ia_3P^1\right)z - 2ia_3P^1 z\ln z\right). \tag{3.21}$$

The only difference from the analysis of the black hole is to keep track of the complex coupling τ that multiplies the H_3 fluxes. The attractor equations tell us, in the limit of small z, that

$$\tau Q_1 - 2ia_3P^1(\ln z + 3/2) \sim 0. \tag{3.22}$$

This leads to an exponentially small modulus $z \sim e^{-Q_1/g_s P^1}$. Actually, we must also turn on an H_3 flux P^0 through the A^0 cycle in order to satisfy the axio-dilaton equation $D_\tau W = 0$ at minimum. This will fix the string coupling as explained in [2].

As argued in [2] this procedure gives a possible explanation for a large hierarchy through the relation between the moduli and the warp factor. The conifold equation is given by

$$y_1^2 + y_2^2 + y_3^2 + y_4^2 = z, \tag{3.23}$$

where a non-zero modulus z cuts off the deep throat. Hence the warp factor can not become arbitrarily small.

We note the similarity with the black hole case. With the particular charges we have chosen the black hole modulus became purely imaginary, while it became real in the flux case (if $\tau = \frac{i}{g_s}$). However, an arbitrary τ yields a complex modulus. Similarly, in the black hole case, a non-zero p^1 charge leads to a complex modulus. This would correspond to the deformed matrix model.

4. A black hole in a flux background

We now come to the main topic of our discussion: a combined analysis where we consider a black hole in a flux background.

There are topological restrictions against introducing D3-branes in backgrounds with fluxes [20 − 23]. Most importantly, the 3-fluxes H_3 and F_3 need to be cohomologically trivial on the world-volume of the brane. This reduces the space of possible charges of the black hole. We will consider completely general charges, and only implicitly assume that they can be consistently introduced into the background in question.

According to the attractor mechanism, complex structure moduli are drawn to fixed values on the horizon of an extremal black hole. This is only true, however, if there is no other contributions to the potential for the complex structure moduli. For instance, in a flux compactification there is a possible conflict with the value determined far away from the black hole through the fluxes. We can expect a competition between the potential as given by the fluxes and the potential induced by the black hole. The physical question we would like to address is whether the black hole, in an appreciable way, can affect where the fluxes lock the moduli.

We imagine a flux compactification where the moduli are fixed at the minimum of $V_f(z)$. That is, we fix $z = z_f$ such that $\partial_z V_f(z_f) = 0$. This remains true even if there is a black hole present provided we are far away from the black hole. What happens if we move in closer? Eventually the black hole potential $V_{bh}(z)$ will start to play a role and we need to consider the combined system.

To exactly solve for a black hole in a flux compactification is certainly a very complicated task. In principle we should start with an ansatz of the form

$$ds^2 = e^{2A(y,\sigma)} \tilde{g}^{(4)}_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y,\sigma)} \tilde{g}^{(6)}_{nm} dy^n dy^m, \tag{4.1}$$

where the four-dimensional part is the same as before and we have allowed for a warp factor depending on space time. We will not go through such an analysis. What we will do instead is simply to estimate when the two competing effects are of comparable order and if and when interesting new physics can occur. To do this we just need the expressions for the respective potentials. The total effective potential piece, ignoring back reaction on the flux term from the black hole piece and vice versa, is given by

$$S_{pot} = -\frac{1}{2\kappa_{10}^2}\int d\mathrm{Vol}_4 \left(\frac{1}{r^4}V_{bh}\left(z\right) + V_f\left(z\right)\right). \tag{4.2}$$

When examining this expression we disregard effects of the warping. Since the warped throat is small compared to the bulk, the warping ought to cancel out when integrating over the whole internal manifold [24].

It is clear from (4.2) that the effect of the black hole is largest at the horizon. There the black hole potential is suppressed by a factor of R^{-4}, where R is the black hole radius. Thus, for any macroscopic black hole, it will be substantially suppressed. In order to study the charge dependence of the suppression we note that for an extremal black hole, the radius is proportional to the charge q of the black hole,[7] $R \sim q$. The potentials themselves are proportional to the square of the corresponding flux quanta Q and charge. We therefore expect that the effect of the black hole is suppressed by a numerical factor $\sim 1/(Q^2q^2)$. Thus the effect on the minimum of the potential should be negligible[8].

This qualitative argument might however go wrong if the functional forms of the two potentials V_f and V_{bh} differ substantially. That this could be the case can be seen from

$$V_f\left(z\right) = e^{K_f}\left(\mathcal{G}_f^{z\bar{z}}D_zW_fD_{\bar{z}}\overline{W}_f + \mathcal{G}_f^{\tau\bar{\tau}}D_\tau W_fD_{\bar{\tau}}\overline{W}_f\right), \tag{4.3}$$

$$V_{bh}\left(z\right) = e^{K_{bh}}\left(\mathcal{G}_{bh}^{z\bar{z}}D_zW_{bh}D_{\bar{z}}\overline{W}_{bh} + |W_{bh}|^2\right), \tag{4.4}$$

where we used the no-scale behaviour to eliminate $-3|W_f|^2$. A simple calculation shows that $\mathcal{G}_f^{\tau\bar{\tau}}D_\tau W_fD_{\bar{\tau}}\overline{W}_f \sim |W_f(\tau \to \bar{\tau})|^2$. Thus, almost identical terms appear in both potentials. The only thing that might be a concern is if the dominant term in $\partial_z V_f$ vanishes.

Let us therefore study these expressions more closely, using the explicit prepotential (3.5). For both the flux and the black hole case let

$$W \sim w(z) = A_0 + A_1 z + A_2 z^2 + A_3 z\ln z \tag{4.5}$$

$$e^{-K} \sim k(z) = B_0 + B_1\left(z-\bar{z}\right)^2 + B_2 z\bar{z}\ln z\bar{z} \tag{4.6}$$

where the A_i and B_i are combinations of flux quanta/charges and geometrical constants, which are linear in the charges. Since we are interested in where the modulus is fixed, we study $\partial_z V$. Using the above expressions the leading terms are[9]

$$\frac{\partial}{\partial z}e^K W\overline{W} \sim \frac{\bar{A}_0}{B_0}\left(A_1 + A_3(\ln z + 1)\right) \tag{4.7}$$

[7] The exact constant of of proportionality depends on the geometry and size of the internal dimensions.

[8] This analysis might not apply to charged black holes that classically have vanishing horizon area. When higher derivative terms in the action are taken into account, these black holes acquire a string scale horizon area that scales as $\sim q$ [25, 26].

[9] Note that these expressions are valid for any charge/flux configuration. In particular, z_{bh} need not lie close to the conifold point.

$$\frac{\partial}{\partial z} e^K \mathcal{G}^{z\bar{z}} D_z W D_{\bar{z}} \overline{W} \sim \frac{1}{z}\left(A_3 \frac{(\bar{A}_1 + \bar{A}_3(\ln \bar{z}+1))}{B_2 \ln z\bar{z}} - \frac{|A_1 + A_3(\ln z + 1)|^2}{B_2\,(\ln z\bar{z})^2}\right). \tag{4.8}$$

We see that (for small z) the dominant contributions to the derivative of the potentials come from $DW\overline{DW}$ in both cases. Let us now add the two potential contributions and solve for z. Since $\ln z\bar{z}$ is a large number we need only consider the first term in (4.8). Thus we wish to solve

$$R^4 \bar{A}_{3f}\,(A_{1f} + A_{3f}(\ln z + 1)) + \bar{A}_{3bh}\,(A_{1bh} + A_{3bh}(\ln z + 1)) = 0. \tag{4.9}$$

Now we use that $(A_{1f} + A_{3f}(\ln z_f + 1)) = 0$ is the zeroth order attractor equation. Thus, we have that

$$A_{1f} + A_{3f}(\ln z + 1) = A_{1f} + A_{3f}(\ln z_f + \ln(z/z_f) + 1) \sim A_{3f} \ln(z/z_f). \tag{4.10}$$

Solving (4.9) now yields

$$\ln z - \ln z_f = -\frac{|A_{3bh}|^2}{|A_{3f}|^2 R^4 + |A_{3bh}|^2}\left(\ln z_f + 1 + \frac{A_{1bh}}{A_{3bh}}\right). \tag{4.11}$$

This equation shows that indeed, for the generic case, z will be fixed close to z_f. This is because the prefactor of the right-hand side is $\sim 1/(Q^2 q^2)$.

However, note that $\ln z_f$ is typically rather large: to create a hierarchy of the weak and Planck energy scales of order $\sim 10^{-15}$ we need $\ln z_f \sim -100$ [2]. Therefore it might suffice to have $1/(Q^2q^2)$ as small as $1/100$ to change the fixed value of the modulus by a factor significantly different from 1. This would correspond to $q \sim 1$, and $Q \sim 10$. If the fluxes and charges are small, this could be possible for a black hole in the end of its evaporation process. For such small charges, however, the supergravity approximation used in this analysis is likely to be invalid.

Performing the same analysis for the case $A_3 = 0$, we obtain the leading contribution to be

$$\partial_z V \sim \frac{1}{z}\frac{|A_1|^2}{B_2(\ln(z\bar{z}))^2}. \tag{4.12}$$

Since, from the attractor equations, the constant $A_{1f} = \mathcal{O}(z_f)$ this contribution is generally much smaller than the dominant term when $A_{3f} \neq 0$. Thus if the black hole has a p^1 charge while there is no P^1 flux we have a problem. This is however a fine-tuned case. For a generic flux compactification such flux will be present.

Our conclusion is that the flux compactifications generically are stable against the introduction of macroscopic black holes. There are some special cases where the black hole might be important: notably if there is no A^1-flux, or at the end of Hawking evaporation provided the fluxes and charges are small. In these cases it would be important to work out the explicit dependence on the geometry of the extra dimensions. We will return to this in a future publication.

5. Conclusions and outlook

In this paper, we have seen that the complex structure moduli stabilization provided by type IIB flux compactifications is stable against the introduction of a charged black hole. We have considered the phenomenologically interesting case when the type IIB theory is compactified on a local deformed conifold, and fluxes are chosen such that the complex structure modulus is fixed near the conifold point. By wrapping D3-branes around cycles of the internal manifold, we have added a black hole to this picture. The leading terms for the moduli-fixing potentials show that the black hole effect is negligible in the generic case. In particular, the black hole contribution is suppressed by $1/(Q^2q^2)$ where Q is integer flux quanta, and q integer black hole charges. We find a few exceptional cases where the conclusion might not hold; for instance if there is no A^1-flux, or at the end of Hawking evaporation provided the fluxes and charges are small.

So far, we have analysed how a black hole influences the moduli fixing of a flux compactification. It would also be interesting to study the reversed question, i.e. how the black hole behaves in a flux background. We have already seen that the black hole attractor mechanism is changed by the fluxes, since these fix the complex structure moduli to a new point in moduli space. Since, for example, the horizon area depends on the value of the moduli at the black hole horizon we might expect that the black hole physics is altered.

Furthermore, our analysis has been qualitative, and quantitative results would be very interesting. To achieve this, the full ten-dimensional equations of motion need to be solved. One would then be in a position to study the moduli fixing exactly and, e.g., how the warping depends on space-time.

We have also seen that, via the internal geometry there is a correspondence between a matrix model and this flux compactification. It would be interesting to see if a matrix model approach could be applied to other aspects of such effective theories. In particular it would be very interesting to investigate whether, as in the black hole case, some matrix model could provide quantum corrections to the compactified theory. Such a relationship could possibly be found via a topological string theory on the generalized Calabi-Yau manifold used in the compactification.

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Note added: while this work was being completed the related paper [27] appeared. There the authors consider a general class of potentials without the restrictions due to flux compactifications which we impose in our work.

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